
Review of Maple 6

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The term *Computer Algebra System* (CAS) is commonly applied to such mathematics packages as Maple, Mathematica, Macsyma, Reduce, Derive and perhaps Mathcad. This reflects their origins as programs for manipulating symbolic expressions for research purposes, rather than being descriptive of what they have become. With their graphical, numerical and presentational capabilities, these packages could now more accurately be described as “Systems for doing, applying and demonstrating Mathematics” – a variant of Stephen Wolfram’s original sub-title of Mathematica. As a consequence of the blending of these separate facilities, the latest CASs are very powerful tools for teaching and learning mathematics.

The package under review, Maple 6, the successor to Maple V release 5.1, is the first product of the collaboration between the Canadian company Waterloo Maple Inc (WMI) and the Oxford based Numerical Algorithms Group (NAG). The rationale behind this particular association seems to be that, give or take a bit of tinkering and bug-fixes, the Maple V series was approaching a state of maturity as far as the symbolics, the worksheet interface and the authoring tools were concerned. Only in the area of numerics was there scope for real innovation and dramatic improvement. This is where the competition is intense; the high-end CASs have all sought to achieve numerical capabilities comparable with those of Matlab. Whilst Wolfram Research has relied on in-house expertise and Macsyma has the Numkit add-on, WMI opted for a collaboration with NAG, the world leader in numerical algorithms. Maple 6 incorporates NAG’s numerical linear algebra routines; those for optimisation and the solution of differential equations etc, are planned for inclusion in later versions. Although such advanced numerics are not of much relevance to this review, I think it is important that readers of this newsletter are at least aware of Maple’s current trajectory.

Maple as a teaching tool

Maple V is well established in many UK Universities as a teaching tool. Indeed, good examples of its uses have often been outlined in this quarterly, in articles and workshop reports. At the University of Liverpool, the use of Maple in the teaching of mathematics to undergraduates has, until recently, been rather limited to certain optional modules and by lecturers’ personal interests. My colleague, David Hodgkinson, has for many years given Maple-based courses in vector calculus to physicists and has also produced material for engineering students. Maple is used in an optional computer-based module “Experimental Mathematics” for second year mathematics students. Some students, tackling optional project essays, have been provided with relevant tuition in the use of Maple. At the M.Sc. level, Maple has for many years been taught in compulsory modules. However, it has never been possible to assume any familiarity with Maple amongst our undergraduate mathematics students, and this has restrained certain imaginative ventures. Happily, this has now changed, with the introduction this academic year of a first year module devoted to IT skills. Under the direction of Dr Alan Irving, and supported by a number of staff and postgraduates as laboratory assistants, the new course covers the following topics: Word processing (MS Word); Using the Web (Netscape); Using spreadsheets for mathematical purpose (MS Excel); Statistics (Minitab); Symbolic mathematics (Maple). The latter is in fact the largest component of this first semester module, and it is designed so that tutors or lecturers teaching contemporaneous and subsequent modules can assume that students have some knowledge of the basics of Maple. Maple is sufficiently user-friendly that students quickly acquire the ability to use it

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at least as a black-box algebraic calculator and for plotting the graphs of functions. Of course, there are surprises and thought provoking concepts in store for the novice, many of which should hopefully help to enhance their understanding of mathematics. For example: the distinction between expressions and functions; the importance of precise and unambiguous mathematical statements; the distinction between lists and sets - the more subtle distinctions between lists and vectors, and between arrays, tables and matrices are best avoided in a first course; the value of thinking of functions both through definitions/algebraic expressions and in terms of a graphical representation; the ambiguities involved in taking roots of numbers, and the related functions.

Beyond the basics, Maple has a role as an exploratory tool. For example, consider functions of the form $x^n \sin(1/x)$. The cases $n=0,1,2$ and 3, are often introduced in calculus courses in the context of function domain/range, continuity and differentiability. A student who is used to producing graphs by the manual plotting of points, will probably have difficulty with the concept of a function which oscillates infinitely often on a finite interval. A guided hands-on approach, using Maple as a plotter and symbolic calculator, is more likely to lead to an understanding of the nature of these functions than a hand-waving blackboard demonstration. In the $n=0$ case, a plot of the function on any interval including the origin, will show a solid blur surrounded by clear bounded oscillations. This does not essentially change as the plotting interval is reduced. Something funny is happening near the origin. On the other hand, a plot whose domain is strictly to the right of the origin, will give a clear picture of how the oscillations are being compressed as we move to the left. The Maple graphics, as shown in figures 1 and 2, are quite convincing.

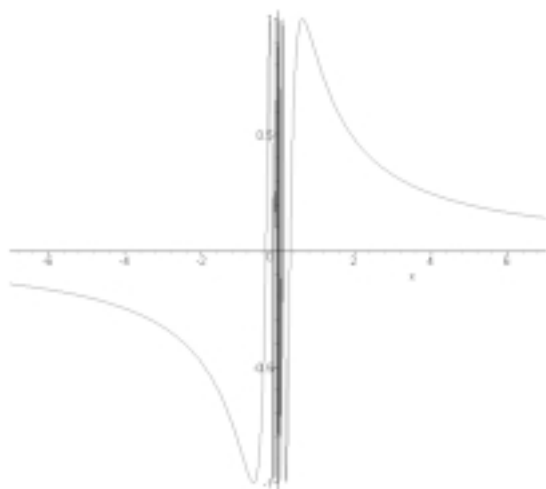


Fig 1

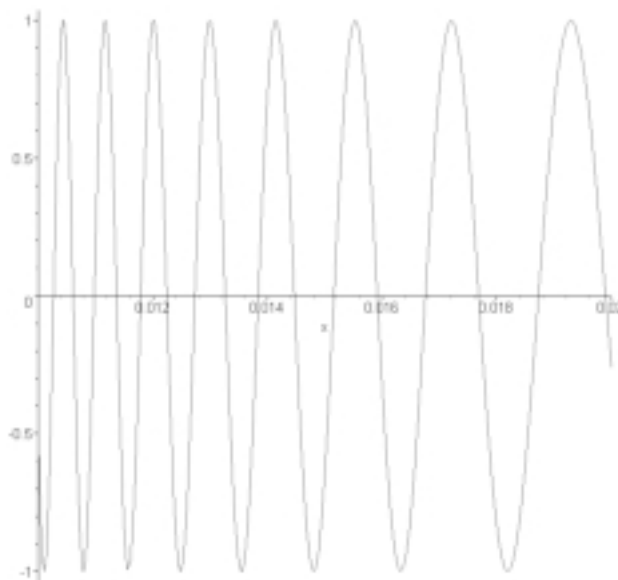


Fig 2

Figure 3 demonstrates the continuity of the $n=2$ case at the origin, using the sandwich theorem. The command to produce this graphic is reproduced below.

```
> plot([x^2*sin(1/x), x^2, -x^2], x=-0.6..0.6, y=-.1..0.1, color=[red,blue,green],
labels=["x-axis", "y-axis"],
labeldirections=[HORIZONTAL, VERTICAL],
legend=["the filling", "upper curve", "lower curve"],
title="The Sandwich Theorem \nA Visual
Demonstration",
titlefont=[TIMES, BOLDITALIC, 14], tickmarks=[3, 2]);
```

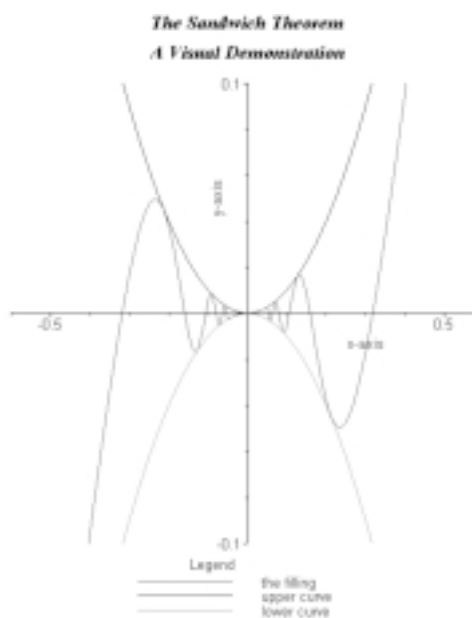


Fig 3

It is instructive to demonstrate graphically that, despite the continuity of the $n=1$ and $n=2$ cases at the origin, neither is differentiable there. In fact $n=3$ is the first everywhere differentiable case. For a second example of a way of introducing an important mathematical point, ask Maple to plot the function $x^{1/3}$ over the interval $-2 < x < 2$. The result is shown in figure 4.

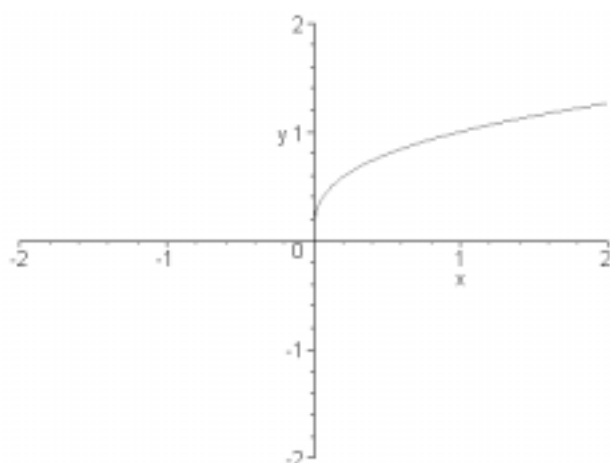


Fig 4

This plot is rather unexpected, for surely every A-level student knows that the cube root of -1 is -1 ; why doesn't Maple acknowledge this well known fact? Of course with a background in complex numbers one realises that the resolution of this problem involves branches of a many-valued complex function. In fact, the input

```
> simplify((-1)^(1/3));
```

has a complex output. If one wants real cube roots of negative numbers, then the function `surd(x,3)` is required. Which brings me to the interesting question of how would the newcomer discover this solution? With great difficulty, I would suggest – `surd` is mentioned, without adequate explanation, in the Learning Guide, and is absent from the index! I say this at the same time as acknowledging that I generally find Maple's help system very good, and I like the supplied manuals.

Another instructive exercise, in the same ball-park, is to consider plots of the functions $\ln(\exp(x))$ and $\exp(\ln(x))$, taken over an interval surrounding the origin. Again, complex numbers resolve the apparently incorrect plot. This example raises a number of other issues, including the concepts of domain/range and of inverse functions. The latter is most easily tackled in Maple by using the plot command in parametric form – plot the pairs $[t, f(t)]$ for the function f , and the pairs $[f(t), t]$ for the inverse.

My final example is concerned with sequences, iteration and convergence. I'm sure we have all asked students how they think an electronic calculator computes square roots. Obviously, the calculator does not actually store the square root of every possible number, so it must utilise some algorithm. I surmise that the built-in method is iterative in nature, which provides a motivation for an investigation of how one might iteratively compute the square root of 2. It is a simple exercise in the theory of 2-term recurrence relations to show that the Fibonacci-like sequence defined by the following Maple procedure ultimately grows by the factor $r = 1 + \sqrt{2}$:

```
> A:=proc(n::nonnegint)
> option remember;
> if n<2 then n+1;
> else 2*A(n-1)+A(n-2);
> end if;
> end proc;
```

Thus successive quotients converge to r .

```
> evalf(seq(A(k)/A(k-1), k=1..8));
2., 2.50000, 2.40000, 2.41667, 2.41379, 2.41429,
2.41420, 2.41422
```

Although general theory tells us that the convergence should be linear, it appears to be relatively fast in this case. A simple analysis, based on the assumption of a linear convergence factor, produces a sequence of estimates of this factor in terms of ratios of differences:

```
> evalf(seq((A(k+2)/A(k+1)-A(k)/A(k-1))/(A(k+1)/
A(k)-A(k)/A(k-1))-1, k=1..6));
-.200000, -.166667, -.172414, -.171429, -.
.171598, -.171569
```

Again general theory tells us that the convergence factor is exactly $(1-\sqrt{2})/(1+\sqrt{2})$, although we don't need to know this to apply the method. It is interesting to try this out on the more familiar quadratically converging iterative scheme defined by the Newton-Raphson scheme for the function $x^2 - 2$:

$$x(n+1) = (x(n) + 2/x(n))/2$$

For a more demanding exercise in the use of symbolics and numerics, it is well worth investigating the application of convergence acceleration schemes, such as Aitken's method.

These are all examples of how Maple can be used to enhance a traditional post-A-level course. I turn now to a consideration of the new features in Maple 6.

What is New in Maple 6?

Although the examples described above can be tackled with any recent version of Maple, previous users will notice some small differences. In particular, fig 3 exhibits the new possibilities for adding legends, multiline titles and vertical axis labelling to plots. There are some other new document processing facilities, such as page numbering, page breaks and page layout, and export to RTF. It is also possible to hide and restore specific types of elements of a Maple worksheet, such as spreadsheets, graphics, Maple input and output – this is not the same as “Remove output” from the Edit menu. At last there is a “find and replace” facility.

Those with experience of writing procedures will have noticed the “end if” rather than “fi” to terminate the if loop in the procedure listed above. “od” has also been replaced by “end do”. Fortunately there are no compatibility problems, as the obsolete loop terminators still work in Maple 6. These trivial cosmetic changes are just part of a big upgrade to the programmability of Maple. A most important new concept is that of the “module”, a far-reaching development of that of the “procedure”. A short example will perhaps give an inkling of what is involved, but barely do it justice. Here is a module:

```
> POWER:=module()
> export c,d;
> local a,b;
> a:=2;
> b:=3;
> c:=x->a^x/b^x;
> d:=x->b^x/a^x;
> end module;
```

```
POWER := module() local a, b; export c, d;
end module
```

The important things here are the “export” variables *c* and *d*, which are available outside of the scope in which they were created. The syntax for accessing them is

```
> POWER:-c(5);
32
-----
243
> POWER:-d(5);
243
-----
32
```

Any further discussion of this would be outside the remit of this review, but it is worth adding that the hard copy Programming Guide has grown in size by 50% to

accommodate the new programming features!

A number of new features with relevance to teaching were added to release 5: palettes; context-sensitive menus; spreadsheets. Each of these has been enhanced. The original palettes were, in my own myopic opinion, microscopic and almost unusable objects. I am delighted to say that large versions are available in Maple 6. There have been welcome improvements to the context-sensitive menus, for example there are now solve and numerically solve menu choices for functions.

There seem to be an abundance of spreadsheet developments. First, the in-built spreadsheet is much better. The package called “spread” allows one to, for example, access cells from a Maple command line. There is the facility to export and import data to and from Matlab, Matrix Market or ascii files. Secondly, it is possible to connect to Microsoft Excel 2000, and use Maple commands and functions therein. Apart from the rather poor (Microsoft) on-line help for the Add-in, and it is another omission from the Learning Guide, this is all very good provided one can think of a useful application. My only use so far has been in the production of tables of functions, derivatives and integrals etc. Perhaps others have found more imaginative uses.

Another new feature in release 5 was the link to Matlab, to access the latter’s powerful numerical routines. This remains, but presumably will tend to become redundant as the results of the collaboration with NAG spread. A brief comment about the latter – it being outside the remit of this review – will hopefully suffice. A new package called LinearAlgebra has been created, along with a whole host of new data structures and syntax, for the numerical and symbolic manipulation of matrices. This package is actually a module.

As always there have been improvements to the mathematical function base and options. In particular, in the areas of differential equations, the handling of discontinuities, integers, quintics and special functions.

I had intended not to mention any bugs in this review. In large systems, bugs are inevitable and developers try hard to eliminate them. Many of the ones I have reported have been fixed, and I have rarely been troubled by them in my own work. That was until a few days ago when I heard through the comp.soft-sys.math.maple news group of a bug in the use of the function “inequal”. Fig 5 shows that the result of the following command is wrong:

```
> with(plots):
inequal({x+y>0,x-y<=2,y<1}, x=-2..3,y=-2..2,
optionsexcluded=(color=yellow),
optionsfeasible=(color=red));
```

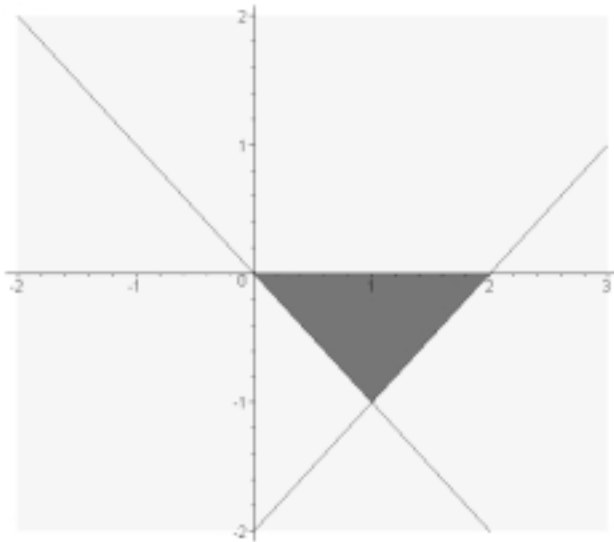


Fig 5

What is particularly irritating is that I happen to want to use this function in a little project I had planned for this Summer! I can't trust Maple 6, and although I am told that "inequal" works OK in release 5.1, can I really be sure?

Conclusions

WMI are basing their marketing campaign for Maple 6 on the success of the NAG collaboration and the related enhancements in programmability. This will have an impact with power users in industry and research, who may well consider migrating from the more expensive Matlab system. That aspect is not so important for the mathematics teaching community. Indeed, I think that this new focus will perhaps divert attention away from the very significant, albeit small improvements that have been achieved in several other areas, some of which were certainly on my wish list.

Response from Scott Rabuka, Product Management Specialist, Waterloo Maple Inc

Thank you for the opportunity to provide feedback on this review of Maple6. Prof Backhouse has positioned Maple 6 very well as a teaching tool, and rightly so! Maple's reputation has long been established in the academic environment.

We are very excited about the partnership with NAG from the UK and the new numerics that have been added to the Maple system. Although we appreciate Prof Backhouse's views on the integration of the NAG Libraries, many professors have expressed a great deal of enthusiasm with the new numerical capabilities in Maple 6. Maple 6 can now be used as a teaching and research tool in a wider range of disciplines relating to numerical Linear algebra.

The review also comments on the add-in for Excel 2000. The purpose of the add-in is to combine the strengths of Excel with the strengths of Maple. Examples of applications for this add-in are available on the Maple Application Center (<http://www.maplesoft.com/apps/index.html> in the What's New section).

Finally, to comment on Prof Backhouses report of the "inequal" bug, it has been reported and we will be investigating it further.

Rest assured that Waterloo Maple Inc is committed to the quality of its product and to the educational market.